A Cylindrical Multiconductor Stripline-Like Microstrip Transmission Line

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***Abstract*—An analytical expression for the Maxwell capacitance matrix of a class of cylindrical inhomogeneous, multiconductor transmission lines is provided. This class includes cylindrical structures symmetrical with respect to the circuit circle and having a symmetry axis. The effective dielectric constant of the line is found to be the arithmetical mean of the dielectric constants of the two media. Hence, this structure enjoys the main advantages of the planar stripline-like microstrip systems. It is also pointed out how to obtain elliptical stripline-like microstrip transmission lines.**

I. Introduction

IN HIS PAPER Garg [1] considered a transmission configuration which has the advantages of both stripline and microstrip line. This new transmission line consists in an inhomogeneous planar shielded structure having equal spacing (substrate thicknesses) between the plane of the circuit and the upper and lower shield planes. In [2] analytical formulas for determining the Maxwell capacitance matrix of multiconductor coupled stripline-like microstrip lines are given. The results obtained confirm that the dominant propagation modes have the same phase velocities independent of the strip widths and spacing and that the velocities are given by the phase velocity of the uncoupled line. This characteristic will result in a very good directivity for multiconductor coupled line directional couplers which use stripline-like microstrip.

Using flexible dielectric materials, it is possible to construct nonplanar transmission lines that can be wrapped around a cylindrical surface. This generates the cylindrical transmission lines which, in the quasi-TEM operating mode, have recently received much attention in the microwave literature. The first approach entails the solution of the Laplace equation in orthogonal curvilinear coordinates (cylindrical or elliptical); in this way Wang [3] analyzed homogeneous cylindrical striplines and microstrip lines, Reddy and Deshpande [4] the cylindrical stripline with multilayer dielectrics, and Joshi and Das [5] homogeneous elliptical striplines. In this method the solution is expressed as an infinite series and the constants are determined by truncating the series; hence the analysis is not rigorous and the solutions are approximate. The second approach [6]–[8] uses conformal mapping of the given structure into a planar one and, further on, uses known results for planar structures [9]. The method is rigorous but in this form it is not suitable for the analysis of cylindrical multiconductor transmission lines. Chan and Mittra [10] applied another approach, the spectral-domain technique [11], to study cylindrical multiconductor transmission lines. This method allows a rigorous numerical iterative procedure which uses the fast Fourier transform algorithm.

This paper presents an analysis leading, in principle, to an exact evaluation of parameters for an extensive class of cylindrical inhomogeneous strip transmission line configurations. The method is based on conformal mapping and on the evaluation of Maxwell's capacitance matrix for the system of aligned planar strips given in [2]. The domain must have a symmetry plane and at the same time must be symmetrical with respect to the circuit surface. In the case of the circular cylindrical structure, this requires that the radius of the circuit cylinder equal the geometrical mean of the radii of the two shield cylindrical surfaces. The assumption that the domain is symmetrical with respect to the circuit surface eliminates most of the complications arising from the inhomogeneity of the dielectric substrate media because the perpendicular component of the electric field at the circuit surface is zero. The effective dielectric constant of the system is simply the arithmetical mean of the dielectric constant values of the two substrate media. Hence, the structure presented here has the principal advantage of the symmetrical plane structure, namely that the mode velocities are independent of the strip widths and spacing; this is why the above structure can be thought of as a cylindrical stripline-like microstrip structure.

It is to be noticed that the above-mentioned symmetry of the structure with respect to a plane is necessary only for computation of the Maxwell capacitance matrix. The independence of the mode velocities of the strip dimensions and spacing also holds if the last symmetry condition is given up. By using certain conformal mappings the above results can be extended to such other nonplanar structures as elliptical stripline-like microstrip structures, which are discussed here as well.

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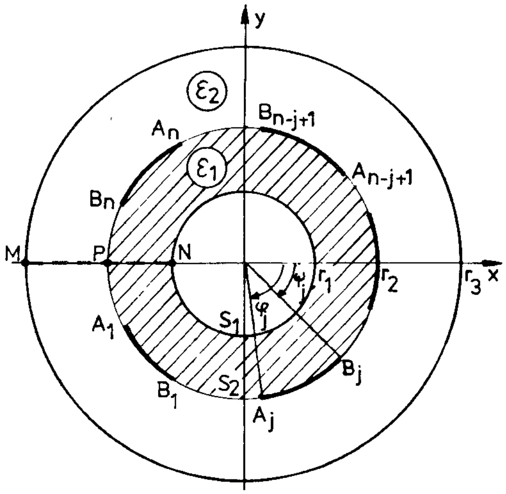

Fig. 1. Multiconductor cylindrical transmission line.

II. Basic Configuration and Conformal Mappings

The cross section of the cylindrical multiconductor transmission line to be analyzed is shown in Fig. 1. It consists of three circles S_1, S_2, S_3; the cylindrical surfaces S_1 and S_3 (of radii r_1 and r_3, respectively) are grounded and the surface S_2 (the circuit surface) separates the two different dielectric media whose relative dielectric constants are ϵ_1 and ϵ_2. On the circuit surface are placed some zero-thickness conducting strips A_jB_j with arbitrary width and spacing but symmetrical with respect to the horizontal plane (Ox axis in Fig. 1). The position of the strips can be characterized by φ_j, ψ_j angles. It is assumed that $r_2 = \sqrt{r_1 r_3}$; i.e., the radius of the circuit surface is the geometrical mean of the shield cylindrical surfaces.

To analyze this system, operating in the quasi-TEM mode, we use two conformal mappings. First, the logarithmic transform

$$z_1 = -i \cdot \ln \frac{z}{r_2} \tag{1}$$

maps the domain bounded by the circles S_1 and S_2 and the line cut into the symmetrical rectangle in the $z_1 = x_1 + iy_1$ plane (Fig. 2).

The logarithm determination in (1) is the principal one and the abscissas of the points A_j, B_j will be φ_j, ψ_j, respectively. Further on, the domain in Fig. 2 is conformally mapped into the $Z = X + iY$ plane with certain line cuts on the real axis (Fig. 3). This is obtained by using a Schwarz–Christoffel transform for representing the upper rectangle (dashed in Fig. 2) in the upper half-plane $\mathrm{Im}\{Z\} > 0$; by symmetry reasons the domain filled with dielectric medium of permittivity ϵ_2 is mapped into the half-plane $Y < 0$. Hence we have

$$Z = \mathrm{sn}(z_1 \cdot K/\pi, k_1). \tag{2}$$

Here sn is the Jacobi function and K is the complete

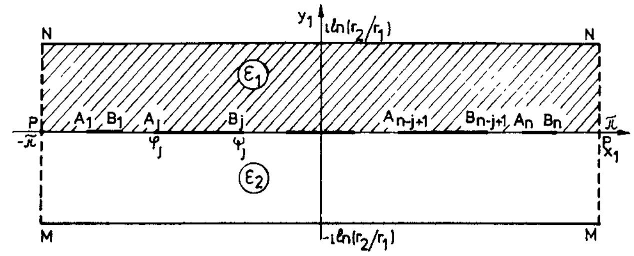

Fig. 2. The planar structure obtained by conformal mapping of the domain in Fig. 1.

elliptic integral of the first kind of modulus k_1. The modulus is the solution of the equation

$$\frac{K(k_1')}{K(k_1)} = \frac{\ln(r_2/r_1)}{\pi}, \qquad k_1' = \sqrt{1-k_1^2}. \tag{3}$$

The abscissas a_j, b_j of the points corresponding to the strip ends can be obtained as

$$a_j = \mathrm{sn}(\varphi_j \cdot K/\pi, k_1) \qquad b_j = \mathrm{sn}(\psi_j \cdot K/\pi, k_1). \tag{4}$$

The symmetry of the domain in Fig. 1 with respect to the Ox axis implies the symmetry of the strips in Fig. 3 with respect to the vertical axis.

III. Determination of Maxwell's Capacitance Matrix

Let V_j, Q_j be, respectively, the potential and the charge on the strip A_jB_j and let $b_0 \equiv b_{n+1}$ be the right end of the first (infinite) conducting strip and a_{n+1} the left end of the other infinite strip in Fig. 3. We use the expression relating the potentials and charges of a system of planar aligned strips given in [2] and [12]:

$$\sum_{k=1}^{n} \sum_{j=0}^{k-1} B(l,j) Q_k = (\epsilon_1 + \epsilon_2) \sum_{k=1}^{n} A(l,k) V_k \qquad (l = 1, \cdots, n) \tag{5}$$

where

$$A(l,k) = (-1)^k \int_{a_k}^{b_k} \frac{t^{l-1}}{\sqrt{|P(t)|}} dt$$

$$B(l,k) = (-1)^{k+1} \int_{a_k}^{b_{k+1}} \frac{t^{l-1}}{\sqrt{|P(t)|}} dt$$

$$P(z) = \prod_{j=1}^{n+1} (z - a_j)(z - b_j). \tag{6}$$

In formula (5) we can take into account the symmetry of the strip. By also using the relation

$$\sum_{j=0}^{n} B(l,j) = 0$$

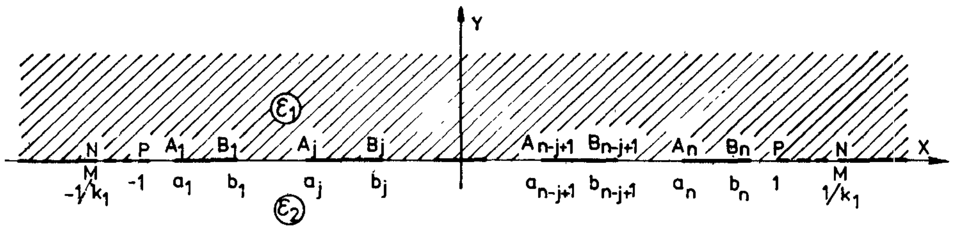

Fig. 3. The canonic domain obtained by conformal mappings of the domain in Fig. 1.

we get

$$\sum_{k=1}^{p}\sum_{j=0}^{k-1} B(l,j)\left[Q_k+(-1)^l Q_{2p-k+1}\right] = (\epsilon_1+\epsilon_2)\sum_{k=1}^{p} A(l,k)\left[V_k+(-1)^l V_{2p-k+1}\right] \quad \text{for } n=2p \text{ (even)} \quad (l=1,\cdots,2p) \tag{7}$$

$$\sum_{k=1}^{p}\sum_{j=0}^{k-1} B(l,j)\left[Q_k+(-1)^{l+1} Q_{2p-k}\right]+Q_p\sum_{j=0}^{p-1}B(l,j) = (\epsilon_1+\epsilon_2)\left\{\sum_{k=1}^{p-1} A(l,k)\left[V_k+(-1)^{l+1} V_{2p-k}\right]+V_pA(l,p)\right\} \quad \text{for } n=2p-1 \text{ (odd)} \quad (l=1,\cdots,2p-1). \tag{8}$$

In relation (8) the coefficient of Q_p and $A(l,p)$ vanishes for even values of the index l.

We now write

$$V_j=V_j^e+V_j^o \qquad Q_j=Q_j^e+Q_j^o$$
$$V_{n-j+1}=V_j^e-V_j^o \qquad Q_{n-j+1}=Q_j^e-Q_j^o$$
$$\left(j=1,\cdots,\left[\frac{n}{2}\right]\right)$$
$$V_{\frac{n+1}{2}}=V^e_{\frac{n+1}{2}} \qquad Q_{\frac{n+1}{2}}=Q^e_{\frac{n+1}{2}} \qquad \text{for } n \text{ odd} \tag{9}$$

and we split the problem into an "even" problem and an "odd" problem.

A. The Even Maxwell Capacitance Matrix

If the potentials of the strips are symmetrical with respect to the Ox axis in Fig. 1, we have $V_j=V_{n-j+1}\equiv V_j^e$; the charges of the strips are symmetrical too, $Q_j=Q_{n-j+1}\equiv Q_j^e$ and the line MN can be assumed to be a magnetic wall. In plane Z the lines MPN are field lines and, consequently, we have

$$b_0^e=b_{n+1}^e=-1/k_1 \qquad a_{n+1}^e=1/k_1$$
$$P^e(Z)=\left(Z^2-k_1^{-2}\right)\prod_{j=1}^{n}(Z-a_j)\cdot(Z-b_j). \tag{10}$$

Relations (7) and (8) become

$$\sum_{k=1}^{p}\left(\sum_{j=0}^{k-1}B^e(2l',j)\right)Q_k^e=(\epsilon_1+\epsilon_2)\sum_{k=1}^{p}A^e(2l',k)V_k^e$$
$$(l'=1,\cdots,p) \quad \text{for } n=2p \tag{11}$$

$$\sum_{k=1}^{p-1}\left(\sum_{j=0}^{k-1}B^e(2l'-1,j)\right)Q_k^e+\frac{1}{2}\left(\sum_{j=0}^{p-1}B^e(2l'-1,j)\right)Q_p^e = (\epsilon_1+\epsilon_2)\left\{\sum_{k=1}^{p-1}A^e(2l'-1,k)V_k^e+\frac{1}{2}A^e(2l'-1,p)V_p^e\right\}$$
$$(l'=1,\cdots,p) \quad \text{for } n=2p-1. \tag{12}$$

Relations (11) and (12) give

$$\boldsymbol{Q}^e=\boldsymbol{C}^e\cdot\boldsymbol{V}^e \tag{13}$$

where the even Maxwell capacitance matrix is

$$\boldsymbol{C}^e=(\epsilon_1+\epsilon_2)[\boldsymbol{N}^e]^{-1}\cdot\boldsymbol{M}^e. \tag{14}$$

The matrices $\boldsymbol{N}^e, \boldsymbol{M}^e$ have the entries

$$N^e(l',k)=\sum_{j=0}^{k-1}B^e(2l',j)$$
$$M^e(l',k)=A^e(2l',k) \qquad (l',k=1,\cdots,p) \tag{15}$$

for $n=2p$ and

$$N^e(l',k)=\sum_{j=0}^{k-1}B^e(2l'-1,j)$$
$$M^e(l',k)=A^e(2l'-1,k) \qquad (k=1,\cdots,p-1)$$
$$N^e(l',p)=0.5\sum_{j=0}^{p-1}B^e(2l'-1,j)$$
$$M^e(l',p)=0.5A^e(2l'-1,p) \qquad (l'=1,\cdots,p) \tag{16}$$

for $n=2p-1$.

B. The Odd Maxwell Capacitance Matrix

In the case where the strip potentials in the physical plane are antisymmetrical with respect to the Ox axis we have $V_j=-V_{n-j+1}\equiv V_j^o$ and the charges of the strips are antisymmetrical too: $Q_j = -Q_{n-j+1} \equiv Q_j^o$. The line MN is now an electric wall at the zero potential and therefore the lines NPM in the Z plane are potential lines. We can write

$$b_0^o = b_{n+1}^o = -1 \qquad a_{n+1}^o = 1$$

$$P^o(Z) = (Z^2-1)\prod_{j=1}^{n}(Z-a_j)\cdot(Z-b_j). \tag{17}$$

Relations (7) and (8) now give

$$\sum_{k=1}^{p}\left(\sum_{j=0}^{k-1} B^o(2l'-1,j)\right)Q_k^o = (\epsilon_1+\epsilon_2)\sum_{k=1}^{p} A^o(2l'-1,k)V_k^o \qquad (l'=1,\cdots,p) \quad \text{for } n=2p \tag{18}$$

$$\sum_{k=1}^{p-1}\left(\sum_{j=0}^{k-1} B^o(2l',j)\right)Q_k^o = (\epsilon_1+\epsilon_2)\sum_{k=1}^{p-1} A^o(2l',k)V_k^o \qquad (l'=1,\cdots,p-1) \quad \text{for } n=2p-1. \tag{19}$$

We have

$$\boldsymbol{Q}^o = \boldsymbol{C}^o\boldsymbol{V}^o \tag{20}$$

where the odd Maxwell capacitance matrix is

$$\boldsymbol{C}^o = (\epsilon_1+\epsilon_2)[\boldsymbol{N}^o]^{-1}\boldsymbol{M}^o. \tag{21}$$

The matrices $\boldsymbol{N}^o, \boldsymbol{M}^o$ are given by relations (22), (23). For $n=2p$ we have

$$N^o(l',k) = \sum_{j=0}^{k-1} B^o(2l'-1,j)$$

$$M^o(l',k) = A^o(2l'-1,j) \qquad (l',k=1,\cdots,p) \tag{22}$$

while for $n=2p-1$,

$$N^o(l',k) = \sum_{j=0}^{k-1} B^o(2l',j) \qquad M^o(l',k) = A^o(2l',k) \qquad (l',k=1,\cdots,p-1). \tag{23}$$

C. The Complete Maxwell Capacitance Matrix

By using the even and odd Maxwell capacitance matrices given above we can now assemble the complete Maxwell matrix of the given structure. We have, for $n=2p$,

$$\begin{aligned} Q_j = Q_j^e + Q_j^o &= \sum_{k=1}^{p}\{C^e(j,k)V_k^e + C^o(j,k)V_k^o\} \\ &= \sum_{k=1}^{p} 0.5[C^e(j,k)+C^o(j,k)]V_k \\ &\quad + \sum_{k=p+1}^{2p} 0.5[C^e(j,2p-k+1) \\ &\quad - C^o(j,2p-k+1)]V_k \qquad \text{for } (j=1,\cdots,p). \end{aligned}$$

Hence

$$C(j,k) = \begin{cases} 0.5[C^e(j,k)+C^o(j,k)] & \text{for } 1 \leqslant k \leqslant p \\ 0.5[C^e(j,2p-k+1)-C^o(j,2p-k+1)] & \text{for } p+1 \leqslant k \leqslant 2p \end{cases} \quad (j=1,\cdots,p)$$

$$C(j,k) = \begin{cases} 0.5[C^e(2p-j+1,k)-C^o(2p-j+1,k)], & 1 \leqslant k \leqslant p \\ 0.5[C^e(2p-j+1,2p-2k+1)+C^o(2p-j+1,2p-k+1)] & \text{for } p+1 \leqslant k \leqslant 2p \end{cases} \quad (j=p+1,\cdots,2p). \tag{24}$$

Similarly in the case where $n=2p-1$ we obtain

$$C(j,k) = \begin{cases} 0.5[C^e(j,k)+C^o(j,k)], & 1 \leqslant k \leqslant p-1 \\ C^e(j,p), & k=p \\ 0.5[C^e(j,2p-k)-C^o(j,2p-k)], & p+1 \leqslant k \leqslant 2p-1 \end{cases} \quad (j=1,\cdots,p-1)$$

$$C(p,k) = \begin{cases} 0.5C^e(p,k), & 1 \leqslant k \leqslant p-1 \\ C^e(p,p), & k=p \\ 0.5C^e(p,2p-k), & p+1 \leqslant k \leqslant 2p-1 \end{cases} \tag{25}$$

$$C(j,k) = \begin{cases} 0.5[C^e(2p-j,k)-C^o(2p-j,k)], & 1 \leqslant k \leqslant p-1 \\ C^e(2p-j,p), & k=p \\ 0.5[C^e(2p-j,2p-k)+C^o(2p-j,2p-k)], & p+1 \leqslant k \leqslant 2p-1 \end{cases} \quad (j=p+1,\cdots,2p-1)$$

We shall now emphasize certain properties resulting from the above relations. Expressions (24) and (25) provide the Maxwell capacitance matrix of the given structure in terms of certain hyperelliptic integrals given by relations (6). The geometry of the physical domain enters these formulas only by the modulus k_1 and the images in the Z plane of the end of the strips; these can be computed by means of relations (3) and (4). The effective determination of the Maxwell capacitance matrix involves some numerical quadrature and some algebra.

Let us now consider the case of the homogeneous dielectric medium ($\epsilon_1 = \epsilon_2 = \epsilon_0$). Relations (13) and (21) give

$$C^e = \frac{\epsilon_1 + \epsilon_2}{2\epsilon_0} C^e_{\text{hom}} \qquad C^o = \frac{\epsilon_1 + \epsilon_2}{2\epsilon_0} C^o_{\text{hom}}$$

where the matrices C^e_{hom}, C^o_{hom} correspond to the homogeneous case. Relations (24) and (25) give

$$C = \frac{\epsilon_1 + \epsilon_2}{2\epsilon_0} C_{\text{hom}}. \tag{26}$$

Therefore, the Maxwell capacitance matrix of the multiconductor system in a stratified dielectric in Fig. 1 can be obtained by relation (26) if we know the matrix of the same multiconductor system in free space. Hence, the effective relative dielectric constant of the structure under consideration is

$$\epsilon_{\text{eff}} = \frac{\epsilon_1 + \epsilon_2}{2\epsilon_0}. \tag{27}$$

This quantity is independent of the strip width and spacing; consequently, this is valid for mode velocities too. Hence the structure shown in Fig. 1 presents the same advantages as stripline-like microstrip structures. It is natural to refer to them as cylindrical stripline-like microstrip structures.

IV. An Elliptical Multiconductor Stripline-Like Microstrip Transmission Line

By using conformal mappings the above results can be extended to other geometries of the stripline. Let us assume, for example,

$$z = \frac{c}{2}\left(z + \frac{1}{z}\right) \tag{28}$$

where $z = \xi + i\eta$ is another complex variable. For $z = re^{i\varphi}$, $0 \leqslant \varphi < 2\pi$, we obtain

$$\xi = \frac{c}{2}\left(r + \frac{1}{r}\right)\cos\varphi$$

$$\eta = \frac{c}{2}\left(r - \frac{1}{r}\right)\sin\varphi, \qquad 0 \leqslant \varphi < 2\pi. \tag{29}$$

These relations give a parametric representation of an ellipse in the z plane of eccentricity c and semiaxes

$$a = \frac{c}{2}\left(r + \frac{1}{r}\right) \qquad b = \frac{c}{2}\left(r - \frac{1}{r}\right). \tag{30}$$

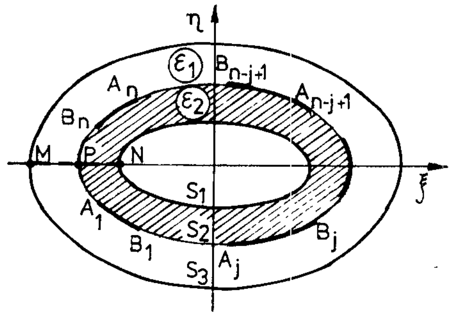

Fig. 4. The multiconductor elliptical transmission line.

The image of the domain plotted in the x plane by the mapping (28) is shown in Fig. 4. The semiaxes of the confocal ellipses S_1, S_2, S_3 are given by relation (30) where $r = r_1, r_2, r_3$, respectively. The positions of the strips on the circuit ellipse are given by relations (29).

Since $r = (a+b)/c$, $c = \sqrt{a^2 - b^2}$, the condition $r_2^2 = r_1 r_3$ implies that

$$\frac{a_2 + b_2}{a_3 + b_3} = \frac{a_1 + b_1}{a_2 + b_2}. \tag{31}$$

Conversely, if we have a transmission line whose cross section contains the three confocal ellipses S_1, S_2, S_3, of semimajor axes a_1, a_2, a_3 and semiminor axes b_1, b_2, b_3 as in relation (31) and symmetrical with respect to the major axes, by using the conformal mapping

$$z = \frac{1}{c}\left(z + \sqrt{z^2 - c^2}\right) \tag{32}$$

we obtain the domain shown in Fig. 1. In the case where the striplines are symmetrical with respect to the minor axes we have to make an additional rotation of $\pi/2$ to map the given domain into the domain considered in Fig. 1. The Maxwell capacitance matrix can be obtained by the formulas given above. Likewise, the mode velocities are independent of the strip sizes and positions on the circuit surface; therefore, this is an elliptical stripline-like microstrip transmission line.

V. Applications

We shall now apply the obtained formulas to two simple structures with one or two coupled striplines. In these cases the given formulas provide analytical relations for the capacitances in terms of elliptic functions.

A. Single-Stripline Case

Since we have $n = 2p - 1$, $p = 1$, we use the relations (14) and (16):

$$N^e(1,1) = 0.5B^e(1,0) = -\frac{1}{2}\int_{-b_0}^{a_1} \frac{dt}{\sqrt{(k_1^{-2} - t^2)(t^2 - a_1^2)}}$$

$$= -0.5k_1 K\left(\sqrt{1 - a^2 k_1^2}\right)$$

$$M^e(1,1) = 0.5A^e(1,1) = -\frac{1}{2}\int_{a_1}^{b_1} \frac{dt}{\sqrt{(k_1^{-2} - t^2)(t^2 - a_1^2)}}$$

$$= -k_1 K(a \cdot k_1).$$

Hence

$$\frac{C^e}{\epsilon_1+\epsilon_2}=\frac{2K(ak_1)}{K\left(\sqrt{1-a^2k_1^2}\right)}. \tag{33}$$

In this case we have

$$\varphi_1=-\varphi \qquad \psi_1=\varphi \qquad b_1=-a_1\equiv a=\operatorname{sn}(\varphi K/\pi,k_1).$$

If $\ln(r_2/r_1)\ll\pi$ and $\varphi\ll\pi$ we can approximate the domain in the z_1 plane by an infinite strip and, correspondingly, the conformal mapping function becomes

$$Z=\tanh\frac{\pi z_1}{2\ln(r_2/r_1)}. \tag{34}$$

In this case we have

$$k_1=1 \qquad a=\tanh\frac{\pi\varphi}{2\ln(r_2/r_1)}$$

$$\sqrt{1-a^2k_1^2}=\operatorname{sech}\left(\frac{\pi\varphi}{2\ln(r_2/r_1)}\right)\equiv s. \tag{35}$$

The characteristic impedance of the line is

$$Z_0=\frac{Z_{0v}}{\sqrt{\epsilon_{\mathrm{eff}}}}\cdot\frac{K(s)}{K(s')} \tag{36}$$

where $Z_{0v}=\sqrt{\mu_0/\epsilon_0}=376.7\ \Omega$ is the characteristic impedance of free space. In the case where $\epsilon_1=\epsilon_2$, relation (36) coincides with that given in [8].

B. Two Equal Striplines

In this case $n=2p$, $p=1$. Let $\psi_2=-\varphi_1=\psi$, $\varphi_2=-\psi_1=\varphi$ be the angles characterizing the ends of the two strips. We have

$$\begin{aligned} &b_2=-a_1=b \qquad a_2=-b_1=a\\ &a=\operatorname{sn}(\varphi K/\pi,k_1)\\ &b=\operatorname{sn}(\psi K/\pi,k_1). \end{aligned} \tag{37}$$

Relations (14) and (15) give

$$\frac{C^e}{\epsilon_1+\epsilon_2}=\frac{M^e(1,1)}{N^e(1,1)}=\frac{K(s_e)}{K(s_e')}, \qquad s_e=k_1\frac{\sqrt{b^2-a^2}}{\sqrt{1-k_1^2a^2}}. \tag{38}$$

Similarly, the odd capacity results by using formulas (21) and (22):

$$\frac{C^o}{\epsilon_1+\epsilon_2}=\frac{M^o(1,1)}{N^o(1,1)}=\frac{K(s_o)}{K(s_o')}, \qquad s_o=\frac{\sqrt{b^2-a^2}}{b\sqrt{1-a^2}}. \tag{39}$$

Relations (24) give the complete Maxwell capacitance matrix in the form

$$\begin{aligned} C(1,1)=C(2,2)&=\frac{1}{2}(C^e+C^o)=\epsilon_{\mathrm{eff}}\left\{\frac{K(s_e)}{K(s_e')}+\frac{K(s_o)}{K(s_o')}\right\}\\ C(1,2)=C(2,1)&=\frac{1}{2}(C^e-C^o)=\epsilon_{\mathrm{eff}}\left\{\frac{K(s_e)}{K(s_e')}-\frac{K(s_o)}{K(s_o')}\right\}. \end{aligned} \tag{40}$$

VI. Conclusions

A class of cylindrical multiconductor transmission lines has been analyzed by using the conformal mapping and the relations between the potentials and the charges of the system of aligned planar strips given in [2]. This class includes cylindrical lines having the radius of the circuit circle as the geometrical mean of the two ground circle radii. In addition, the system exhibits a symmetry axis.

The paper provides an analytical expression of the Maxwell capacitance matrix in terms of hyperelliptic integrals. The capacitance matrix of the multistrip system in a stratified dielectric is expressed by relation (26) in terms of the Maxwell capacitance matrix of the same system placed in free space. Since the effective dielectric constant of the structure is simply the arithmetical mean of the dielectric constants of the two substrate media, the mode velocity is constant and therefore the considered cylindrical structure enjoys the properties of planar stripline-like microstrip structures. It is also pointed out how these results can be extended to elliptical multiconductor transmission lines.

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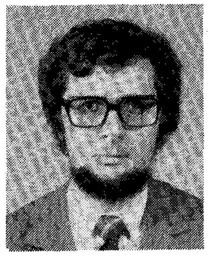

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